# AMS Foundation Exam (January 2022): Probability Questions

Date of Exam: January 20, 2022

Time: 11:15 am - 1:15 pm EST

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

**NAME:** _____ **ID:** _____

**Signature:** _____

**Instruction page**

There are 3 problems, and you are required to solve all of them. All problems are weighted equally. Please show detailed work for full credit.

- This exam is conducted via Zoom on January 20 from 11:15 am to 1:15 pm EST.

- The entire Zoom meeting and chat messages are being recorded.

- This is a closed book, closed note exam.

- Hand calculators (or other computing devices) may not be used during the exam.

- You should join the Zoom meeting from two devices: Your computer/laptop/tablet (with webcam), and your smartphone (with camera).

- Audio should be muted and video must be kept on during the exam.

- Your computer webcam must fully show your face; your smartphone camera should show your computer monitor, your hands and workspace, with the pages of paper being used for the exam.

- At the very beginning of the exam, during set up, you will be asked to do a brief "environment scan", showing the workspace where your computer is and the desk/table/floor where you will be writing your work.

- You are required to bring enough blank pieces of paper to use for the exam. You will show the blank pages at the beginning, during the "environment scan" on Zoom.

- You are not allowed to use the internet for any searching or communication with others, with the sole exception of communication privately with the proctors via Zoom chat (which is set so that your chats only go privately to hosts, not to others).

- It is recommended that you print the exam and write your answers on it. However, you can write your answers on your blank papers if you do not have a printer with you.

- After you finish the exam, scan your pages, ordered and oriented appropriately, into a single pdf file. Email the pdf file to
  **eugene.feinberg@stonybrook.edu** and cc to **hongshik.ahn@stonybrook.edu**
  no later than 5 minutes after completion of the exam (i.e., **by 1:20 pm EST**).

- No students are allowed to leave the Zoom meeting until the exam is over.

  – If you finish the exam early, then submit your exam and remain in the Zoom meeting until the conclusion of the exam at 1:15 pm EST.

  – After submitting your exam, you can study for another exam or work on anything else, while staying in view in the Zoom meeting.

- If the answers are not submitted by 1:20 pm EST, the exam will not be graded, and a score of zero will be given.

- If you have a question during the exam, then send a chat message to the host privately.

1. Let $X$ be uniform on $(-\pi, \pi)$, and let $Y = a\tan(X)$, $a > 0$. Find the probability density function $f_Y(y)$ of $Y$.

2. Let $X$ be a random variable with mean $\mu$ and variance $\sigma^2$, both finite. Show that for any $b > 0$
$$P\{X \geq \mu + b\sigma\} \leq \frac{1}{1 + b^2}.$$

3. Let $(A_\gamma)_{\gamma \in \Gamma}$ be an infinite family of mutually exclusive events. This means that the following properties hold: (i) $\Gamma$ is an infinite set, (ii) each $A_\gamma$, $\gamma \in \Gamma$, is a subset of the sample space, for which the probability $P(A_\gamma)$ is defined, and (iii) $A_{\gamma_1} \cap A_{\gamma_2} = \emptyset$ for all $\gamma_1, \gamma_2 \in \Gamma$ such that $\gamma_1 \neq \gamma_2$. Prove that, if $P(A_\gamma) > 0$ for all $\gamma \in \Gamma$, then $\Gamma$ is a countable set.