# Mathematical Statistics Qualifier Examination
## (Part I of the STAT AREA EXAM)
## May 23, 2018; 9:00AM – 11:00AM

Name: ________________________________________________ ID: ____________________________ Signature: ____________________________

Instruction: There are 4 problems – you are required to solve them all. Please show detailed work for full credit. This is a close book exam from 9am to 11am. You need to turn in your exam by 11am, and subsequently, receive the questions for your applied statistics exam. Please do NOT use calculator or cell phone or smart watch. Please show complete procedures and simplify the results as best as you can for full credit. Good luck!

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from the truncated exponential distribution with pdf: $f(x) = \exp\left[-\left(x-\theta\right)\right],\ \textit{if}\ x \ge \theta;\ \textit{and}\ f(x) = 0,\ \textit{if}\ x < \theta$. Please derive the $100(1-\alpha)\%$ confidence interval for θ by (a) inverting the two-sided likelihood ratio test, (b) using the pivotal quantity method. Furthermore, please (c) compare these two intervals.

2. Let $X_1, X_2, \cdots, X_{n_1} \overset{iid}{\sim} N(\mu_1, \sigma^2)$, $Y_1, Y_2, \cdots, Y_{n_2} \overset{iid}{\sim} N(\mu_2, \sigma^2)$, be two independent samples with all parameters unknown. Please derive the likelihood ratio test of $H_0: \mu_1 - \mu_2 = 0\ \ vs\ H_1: \mu_1 - \mu_2 > 0$. Please discuss whether this is a uniformly most powerful test or not.

3. A coin, believed to be biased, is tossed until the first head appears. Let $\theta$ be the chance the coin lands heads. Let $x_i$ be the number of tails before the first head so that $x_i$ can take values 0, 1, 2, ... This procedure is repeated n = 5 times and $\sum_{i=1}^{5} x_i = 17$. Suppose the prior for $\theta$ is taken as a mixture of two beta priors
$$p(\theta) = \frac{1}{2} \cdot \frac{\Gamma(12)}{\Gamma(3)\Gamma(9)} \theta^2 (1-\theta)^8 + \frac{1}{2} \cdot \frac{\Gamma(12)}{\Gamma(9)\Gamma(3)} \theta^8 (1-\theta)^2$$
Please find the posterior distribution of $\theta$.

4. Let $X_1, X_2, \cdots, X_n$ be a random sample with probability density function (pdf)
$$f(x;\theta) = \theta exp(-\theta x), \qquad x \ge 0,\ \ \theta > 0$$
(a) Please show that $\frac{n}{\sum_{i=1}^{n} X_i}$ is a consistent estimator of $\theta$.
(b) Please derive the asymptotic distribution of $\frac{n}{\sum_{i=1}^{n} X_i}$.
(c) Please derive the Wald asymptotic size $\alpha$ test for $H_0: \theta = \theta_0$ versus $H_a: \theta \neq \theta_0$

*** That's all, folks! ***