Sunday, November 8th, 5:30 pm - 6:15 pm, Invited Talk

Sparse Sampling Methods for Experimental Data from the DOE Facilities

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Abstract
This talk will focus on mathematics developed by the ACUMEN project to help with the mathematical challenges faced by the DOE at the experimental facilities. This talk will discuss sparse sampling methods and fast optimization developed specifically for image processing and data analysis. Sparse sampling has the ability to provide accurate reconstructions of data and images when only partial information is available for measurement. Sparse sampling methods have demonstrated to be robust to measurement error. These methods have the potential to scale to large computational machines and analysis large volumes of data. In recent years sparse sampling methods has received considerable attention for designing image reconstruction algorithms from under-sampled and noisy data for images that have some sparsity properties, that is, some measurable features of the image have sparse representation. Technically, the best measure of sparsity is the l0 norm. However, the l0 norm does not meet the convexity requirements and is very slow to compute, so l1 regularization is used as a substitute [4]. It has been demonstrated in many studies that l1 regularization provides a formulation that is compatible with compressed sensing (CS) applications, specifically, when an image can be reconstructed from a very small number of measurements [1, 3]. In particular, the goal for reconstructing a sparse representation of an image sampled in the Fourier domain is to solve

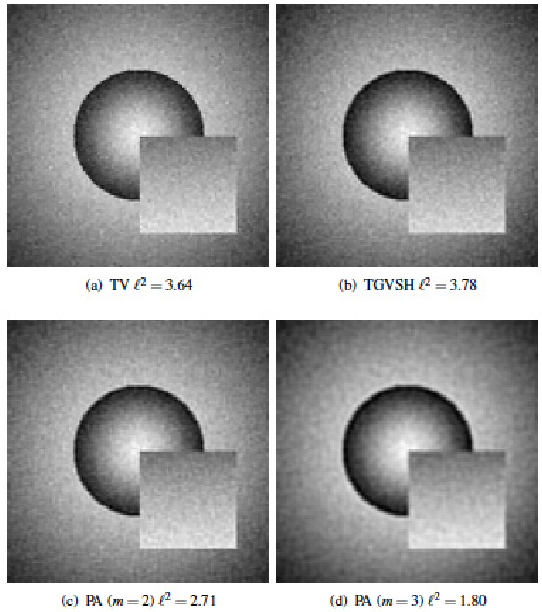

Figure 1: Reconstruction of $f_c(x,y)$ given noisy Fourier data (10 dB SNR) and using tomographic sampling.

$$\min_{\mathbf{f}} J(\mathbf{f}) \text{ such that } ||\mathcal{F}\mathbf{f} - \hat{\mathbf{f}}||_2 = 0,$$

where $\hat{f}$ consists of samples of the Fourier transform of the unknown image, f. F contains a subset of rows of a Fourier matrix, and J is an appropriate l1 regularization term, [5, 8, 9]. Typically for measured data the related (TV) “denoising” problem,

$$\min_{f} J(\mathbf{f}) \text{ such that } ||\mathcal{F} f - \hat{\mathbf{f}}||_2 < \sigma,$$

is solved. It is in general still difficult to develop efficient and robust techniques for solving (2). The Split Bregman Algorithm, [6], is a numerically efficient and stable algorithm that has successfully solved (2) for a variety of applications. In this work we use the Split Bregman Algorithm as a launching point to develop a new technique for solving (2) based on the polynomial annihilation l1 regularization, [11, 12]. We demonstrate that our method yields improved accuracy in regions away from discontinuities, especially in the case of under-sampled data. We will adopt the standardizations and terminology from [6] to describe our algorithm. To illustrate algorithm we consider the following test function defined on [–1;1]2:

$$f_c(x,y) = \begin{cases} \sin(\pi\sqrt{x^2+y^2}/2) & \text{if } 0 < x,y < \frac{3}{4} \\ g(x,y) & \text{otherwise,} \end{cases}$$

$$g(x,y) = \begin{cases} \cos(3\pi\sqrt{x^2+y^2}/2) & \text{if } \sqrt{x^2+y^2} \leq \frac{1}{2} \\ \cos(\pi\sqrt{x^2+y^2}/2) & \text{if } \sqrt{x^2+y^2} > \frac{1}{2}. \end{cases}$$

Figure 1 compares the results for reconstructing fc(x;y) using the same techniques for the case where the Fourier data are sampled using the tomographic pattern with noise level 10dB SNR. It can be seen that using high orders of the polynomial annihilation l1 regularization reduces the error in reconstruction.

References

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Biography

Richard K. Archibald is a staff scientist in the Computational Mathematics group at ORNL. He held the Alston S. Householder Fellowship in Scientific Computing from 2005 until his staff appointment in 2007. Archibald received both his BS in physics and his MS in mathematics from the University of Alberta in Edmonton, Canada in 1996 and 1998, respectively. He obtained his PhD in mathematics from the University of Alberta-Edmonton in 2002.

Archibald's current research interests include designing algorithms for the next generation of high-performance architecture, establishing long-time integration steps for the dynamical core of climate models at high resolution, and mathematical challenges for Neutron science.